Geometry 8.1 – Geometric Mean and 8.2 Pythagorean Theorem NOTES

KeyConcept Geometric Mean

Words: The geometric mean of two positive numbers \( a \) and \( b \) is the number \( x \) such that \( \frac{a}{x} = \frac{x}{b} \). So, \( x^2 = ab \) and \( x = \sqrt{ab} \).

Example: The geometric mean of \( a = 9 \) and \( b = 4 \) is 6, because \( 6 = \sqrt{9 \times 4} \).

Example 1: Geometric Mean

Find the geometric mean for each problem.

1) 8 and 10
   \[
   \frac{8}{x} = \frac{x}{10} \\
   x^2 = 80 \\
   x = \sqrt{80} \\
   x = 8.94
   \]

2) 5 and 45
   \[
   \frac{5}{x} = \frac{x}{45} \\
   x^2 = 225 \\
   x = 15
   \]

3) 12 and 15
   \[
   \frac{12}{x} = \frac{x}{15} \\
   x^2 = 180 \\
   x = \sqrt{180} \\
   x = 13.42
   \]

Pythagorean Triple: a set of three nonzero whole numbers \( a \), \( b \), and \( c \) such that \( a^2 + b^2 = c^2 \)

KeyConcept Common Pythagorean Triples

<table>
<thead>
<tr>
<th>3, 4, 5</th>
<th>5, 12, 13</th>
<th>8, 15, 17</th>
<th>7, 24, 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>6, 8, 10</td>
<td>10, 24, 26</td>
<td>16, 30, 34</td>
<td>14, 48, 50</td>
</tr>
<tr>
<td>9, 12, 15</td>
<td>15, 36, 39</td>
<td>24, 45, 51</td>
<td>21, 72, 75</td>
</tr>
<tr>
<td>3x, 4x, 5x</td>
<td>5x, 12x, 13x</td>
<td>8x, 15x, 17x</td>
<td>7x, 24x, 25x</td>
</tr>
</tbody>
</table>

Example 2: Use the Geometric Mean with Right Triangles

4) Find \( x \), \( y \) and \( z \).

\[
\frac{5}{x} = \frac{x}{20} \\
x = \sqrt{100} \Rightarrow x = 10
\]

\[
\frac{5}{y} = \frac{y}{25} \\
y = \sqrt{125} \Rightarrow y = 11.2
\]

\[
\frac{20}{z} = \frac{z}{25} \\
z = \sqrt{500} \Rightarrow z = 22.4
\]
5) Find \(x, y\) and \(z\).

\[
\frac{8}{z} = \frac{z}{25} \\
z^2 = 200 \\
z = 14.1
\]

\[
\frac{8}{x} = \frac{x}{33} \\
x^2 = 264 \\
x = 16.2
\]

\[
\frac{25}{y} = \frac{y}{33} \\
y^2 = 825 \\
y = 28.7
\]

6) Find \(x, y\) and \(z\).

\[
\frac{25}{z} = \frac{z}{9} \\
9x = 144 \\
x = 16
\]

\[
\frac{9}{z} = \frac{z}{25} \\
z^2 = 225 \\
z = 15
\]

\[
\frac{16}{y} = \frac{y}{25} \\
y^2 = 400 \\
y = 20
\]

**Pythagorean Theorem**

**Theorem 8.4 Pythagorean Theorem**

Words: In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Symbols: If \(\triangle ABC\) is a right triangle with right angle \(C\), then \(a^2 + b^2 = c^2\).

**Example 3: Find missing measures using the Pythagorean Theorem**

Find the missing variable for each problem.

7) \(a^2 + b^2 = c^2\) \\
\(6^2 + 15^2 = x^2\) \\
\(36 + 225 = x^2\) \\
\(x = \sqrt{261}\) \\
\(x = 16.2\)
Example 4: Use the Pythagorean Theorem

9) Damon is locked out of his house. The only open window is on the second floor, which is 12 feet above the ground. He needs to borrow a ladder from his neighbor. If he must place the ladder 5 feet from the house to avoid some bushes, what length of ladder does Damon need?

10) According to your company’s safety regulations, the distance from the base of a ladder to a wall that it leans against should be at least one fourth of the ladder’s total length. You are given a 20 foot ladder to place against a wall at a job site. If you follow the company’s safety regulations, what is the maximum distance $x$ up the wall the ladder will reach, to the nearest tenth?
Pythagorean Inequality Theorems

**Theorems**  

**8.6** If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is an acute triangle.  

**Symbols**  

If \( c^2 < a^2 + b^2 \), then \( \triangle ABC \) is acute.

**8.7** If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is an obtuse triangle.  

**Symbols**  

If \( c^2 > a^2 + b^2 \), then \( \triangle ABC \) is obtuse.

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**Example 5: Classify Triangles**

Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as acute, right or obtuse. Justify your answer.

11) 11, 60, 61  
\[ 11 + 60 > 61 \checkmark \]  
\[ 11^2 + 60^2 = \text{not equal to} \ 61^2 \]  
\[ 11 + 61 > 60 \checkmark \]  
\[ 121 + 3600 > 3721 \]  
\[ 3721 = 3721 \]  
\[ \text{Right} \]

12) 7, 14, 16  
\[ 7 + 14 > 16 \checkmark \]  
\[ 14 + 16 > 7 \checkmark \]  
\[ 7 + 16 > 14 \checkmark \]  
\[ 49 + 196 \checkmark 256 \]  
\[ 244 \checkmark 256 \]  
\[ \text{OBTUSE} \]

13) 6.2, 13.8, 20  
\[ 6.2 + 13.8 \checkmark 20 \]  
\[ \text{Not a triangle} \]

14) \( 2\sqrt{3} \), \( 4\sqrt{2} \), \( 3\sqrt{5} \)  
\[ 2\sqrt{3} + 4\sqrt{2} \checkmark \]  
\[ 3\sqrt{5} \]  
\[ 3.5 + 5.7 \checkmark 6.7 \]  
\[ 5.7 + 6.1 \checkmark 3.5 \]  
\[ 3.5 + 6.1 \checkmark 5.7 \]  
\[ \text{OBTUSE} \]